Optical refractometry based on Fresnel diffraction from a phase wedge

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Received August 9, 2010; revised October 7, 2010; accepted October 7, 2010;
posted October 11, 2010 (Doc. ID 132985); published October 28, 2010

A method that utilizes the Fresnel diffraction of light from the phase step formed by a transparent wedge is introduced for measuring the refractive indices of transparent solids, liquids, and solutions. It is shown that, as a transparent wedge of small apex angle is illuminated perpendicular to its surface by a monochromatic parallel beam of light, the Fresnel fringes, caused by abrupt change in refractive index at the wedge lateral boundary, are formed on a screen held perpendicular to the beam propagation direction. The visibility of the fringes varies periodically between zero and 1 in the direction normal to the wedge apex. For a known or measured apex angle, the wedge refractive index is obtained by measuring the period length by a CCD. To measure the refractive index of a transparent liquid or solution, the wedge is installed in a transparent rectangle cell containing the sample. Then, the cell is illuminated perpendicularly and the visibility period is measured. By using modest optics, one can measure the refractive index at a relative uncertainty level of 10^{-5}. There is no limitation on the refractive index range. The method can be applied easily with no mechanical manipulation. The measuring apparatus can be very compact with low mechanical and optical noises.

OCIS codes: 120.3940, 120.3930, 260.1960.

The refractive index is an important characteristic parameter of a matter. Therefore, a large number of methods has been introduced for its measurement. Most of these methods can be classified in three distinct categories. The methods in the first category, which are based on the measurement of the refraction angle by ray optics methods, include the commonly used refractometers named after Abbe, Pulfrich, and Hilger–Chance [1–4]. The accuracy of these methods depends on the precision with which the deviation angle is measured. High-precision measurement requires high-quality optics and precise measurement of the deviation angle. For small deviation angles, the moiré technique has also been used for refractive index measurement [5]. In the methods of the second category, the interference of light has been utilized. The optical path difference depends on the refractive indices of the media light is traveling through. Measuring the path length and optical path by an interferometer provides the refractive index of the medium. Different interferometers, including those associated with Michelson, Mach–Zehnder, Fissu, and Jamin, have been used for this purpose [6–10]. Interferometers are adequate devices for measuring the refractive index change. However, interferometry requires fine optics and it is sensitive to mechanical and optical noises. Recently, diffraction from gratings and diffraction combined with interference have been used for measuring the refractive indices of liquids [11,12]. The diffraction angle of light diffracted from a grating depends on the refractive index of its surrounding medium. This is the base for the methods in the third category.

The presented refractometry is based on the Fresnel diffraction of light from the neighborhood of the lateral interface of a transparent wedge and the measurement criterion is the fringe visibility change. The method is applicable to solids, liquids, and gases. It imposes no limitation on the refractive index range. It can be applied easily and, with modest equipment, provides the refractive index at a relative uncertainty level of 10^{-5}. Mechanical manipulation is not required during the measurement process. The measuring apparatus can be very compact with very low mechanical and optical noises.

The Fresnel diffraction from phase steps was recently systematically studied and formulated [13–16]. According to these studies, when a part of a spatially coherent beam of light experiences an abrupt change in phase, the intensity redistributes across the beam reflected or transmitted from the affected part. The intensity redistribution can be described adequately by the Fresnel diffraction of light from the neighboring area of the region with the abrupt phase change. An abrupt phase change can be imposed on a part of a coherent beam by reflecting it from a physical step [Fig. 1(a)], or transmitting it through a transparent plate of refractive index N immersed in a liquid or gas of refractive index N' [Fig. 1(b)]. In both cases, there is an abrupt change in the phase at the edges. The typical diffraction patterns and corresponding intensity profiles are shown in Figs. 2(a) and 2(b), respectively.

The diffracted intensity at point P in Fig. 1(a), which is located on specularly reflected ray P_0P, depends on the location of P_0. For a step with the same reflectance on both sides of the step edge, the normalized diffracted intensities for P_0 on the left and right sides of the step edge, specified by $-$ and $+$, are given as [14]

$$I_n = \cos^2\left(\frac{\phi}{2}\right) + 2(C_0^2 + S_0^2)\sin^2\left(\frac{\phi}{2}\right) \mp (C_0 - S_0)\sin\phi, \quad (1)$$

where

$$\phi = \frac{4\pi h}{\lambda}\cos\theta \quad (2)$$

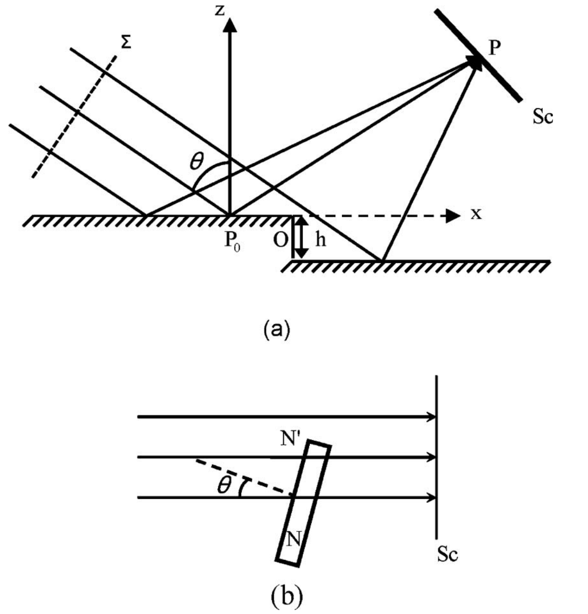

Fig. 1. (a) Plane wave Σ is incident to a one-dimensional (1D) phase step of height h. (b) A transparent plate of refractive index N immersed in a medium of refractive index N' provides a step in transmission.

is the phase change that is imposed by the step of height h at incident angle θ on a light of wavelength λ. The parameters C_0 and S_0 are the Fresnel cosine and sine integrals associated to the distance P_0O in Fig. 1(a). For a step in transmission [Fig. 1(b)] the intensity is expressed by Eq. (1), but the phase change is the following:

$$\phi = \frac{2\pi}{\lambda} N'h \left[\sqrt{n^2 - \sin^2\theta} - \cos\theta \right], \tag{3}$$

where $n = \frac{N}{N'}$ and h is the thickness of the plate. According to Eq. (1), for $\phi = 2m\pi (m = 0, \pm 1, \pm 2, \ldots)$, we get $I_n = 1$ across the diffraction pattern, which means fringes with zero visibility. For $\phi = (2m+1)\pi$ and $C_0 = S_0 = 0$, which corresponds to the step edge, the

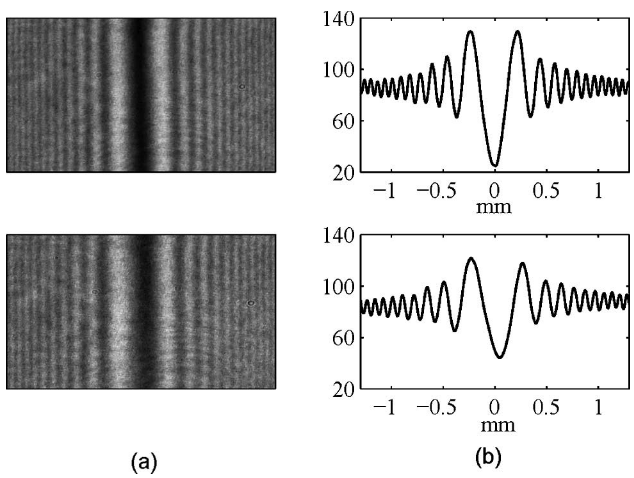

Fig. 2. Fresnel diffraction patterns of light diffracted from two 1D phase steps of different heights and the corresponding intensity profiles. As the profiles show, the visibility of the fringes varies with the step height. The step edge is a line parallel to the fringes that passes from point 0 on the profiles.

intensity is zero, and therefore, the visibility of the three central fringes (upper pattern in Fig. 2), becomes equal to 1. Thus, the visibility of the diffraction fringes are a periodic function of ϕ that can be varied by changing the angle of incidence or the thickness of the plate. Now, by replacing the plate in Fig. 1(b) with a wedge of a small angle α (Fig. 3) we get a plate that imposes a linearly varying phase in the y direction. The thickness of the wedge at distance y from the wedge apex is $y \tan\alpha$. Considering the latter in Eq. (3), for normal illumination of the wedge, we get

$$\phi = \frac{2\pi y}{\lambda}(N - N') \tan\alpha. \tag{4}$$

Thus, the visibility of the diffraction fringes varies periodically in the y direction. By changing ϕ by 2π, we get the period length ρ:

$$\rho = \frac{\lambda}{(N - N') \tan\alpha}. \tag{5}$$

The diffraction fringes of the wedge are normal to the wedge vertex, and the periodicity of the visibility occurs in the fringe direction. Therefore, any change of the fringe spacing due to the change of wedge–screen distance does not affect the period length ρ. Thus, the accuracy in the measurement of N or N', when α and N or N' are known, depends only on the precision with which the period ρ is measured.

To measure the refractive index of a transparent object, the object is prepared in a wedge form with flat surfaces and small apex angle. First, the apex angle is measured accurately. Then, the wedge is illuminated perpendicularly with an expanded monochromatic parallel beam, as shown in Fig. 3. The diffraction fringes are recorded by a CCD connected to a computer. Measuring

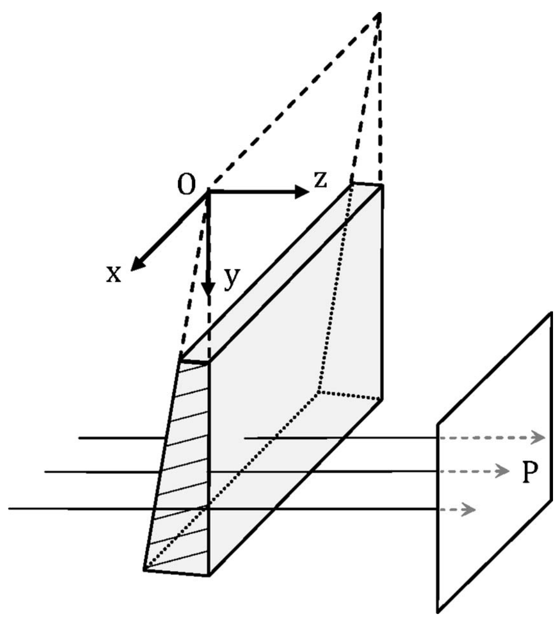

Fig. 3. When a transparent wedge of small apex angle is illuminated by a monochromatic parallel beam, the light diffracted from the two sides of the hatched interface produce a diffraction pattern similar to those shown in Fig. 4.

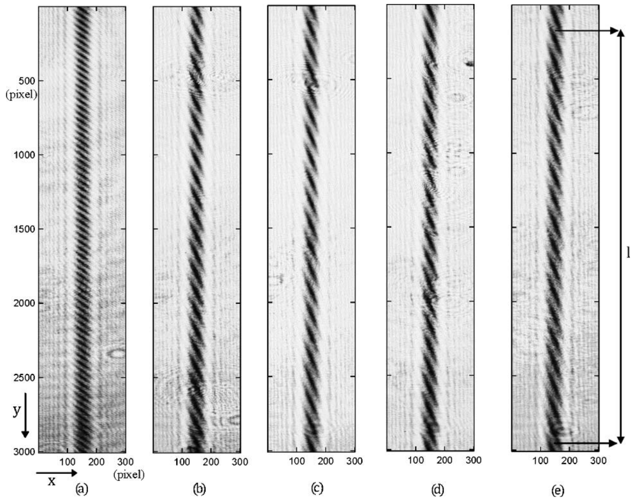

Fig. 4. Diffraction patterns of the expanded He–Ne laser light diffracted from a BK7 wedge of apex angle $\alpha = 0.198°$, immersed in (a) air, (b) water, (c) acetone, and in ethanol–water solutions of (d) 30% and (e) 80%. The slanted fringes at the center are the traces of the central dark fringe in Fig. 2, whose location and intensity vary periodically by the change of the wedge thickness. The arrows represent the directions of the x and y axes in Fig. 3.

the distance between two similar visibility states at far ends, and dividing the distance by the number of periods in between, the period ρ is obtained. Having N', namely, the air refractive index, from Eq. (5) we get

$$N = N' + \frac{\lambda}{\rho \tan \alpha}. \qquad (6)$$

For given N', λ, and α, the measurement uncertainty in N, $U(N)$, can be expressed as

$$U(N) = \frac{\lambda}{\rho \tan \alpha} \frac{U(l)}{l}, \qquad (7)$$

where l is the distance between two similar visibility states (Fig. 4) and $U(l)$ is the uncertainty in l.

The Fresnel diffraction pattern shown in Fig. 4(a) was obtained by illuminating a BK7 wedge of apex angle $\alpha = 0.198°$ perpendicularly by an expanded He–Ne laser beam of diameter 60 mm and recording the fringes with a CCD of pixel size $7.8\ \mu m \times 7.8\ \mu m$. We used two BK7 wedges of size $15\ mm \times 30\ mm$ and apex angles $\alpha_1 = 0.198°$ and $\alpha_2 = 0.064$ and obtained $N = 1.5152 \pm 2 \times 10^{-4}$ and $N = 1.5151 \pm 2 \times 10^{-4}$ for $\lambda = 632.8$ nm.

For measuring the refractive indices of the liquids and solutions, a BK7 wedge of apex angle $\alpha = 0.198°$ and size $15\ mm \times 30\ mm$ was installed in a transparent rectangular cell of dimensions $25\ mm \times 36\ mm \times 36\ mm$ that contained the sample. The cell was illuminated with an expanded He–Ne laser beam in the described manner. The typical diffraction patterns obtained for two different liquids and two solutions of different concentrations are shown in Figs. 4(b)–4(e). As can be seen from the patterns, the diffraction fringes are vertical, perpendicular to the wedge vertex, but their locations change with the wedge thickness. The refractive indices of the three liquids that were measured at 21 °C are presented in Table 1. The plot in Fig. 5 represents the refractive index of an ethanol–water solution versus an ethanol concentration obtained by this technique.

This study shows that optical refractometry based on Fresnel diffraction from a phase wedge is a highly accurate, reliable, and easily applicable technique with no limitation on refractive index range.

Table 1. Refractive Indices of Three Different Liquids

Liquid	Refractive Index N
Water	$1.3317 \pm 2 \times 10^{-4}$
Acetone	$1.3578 \pm 2 \times 10^{-4}$
Ethanol	$1.3608 \pm 2 \times 10^{-4}$

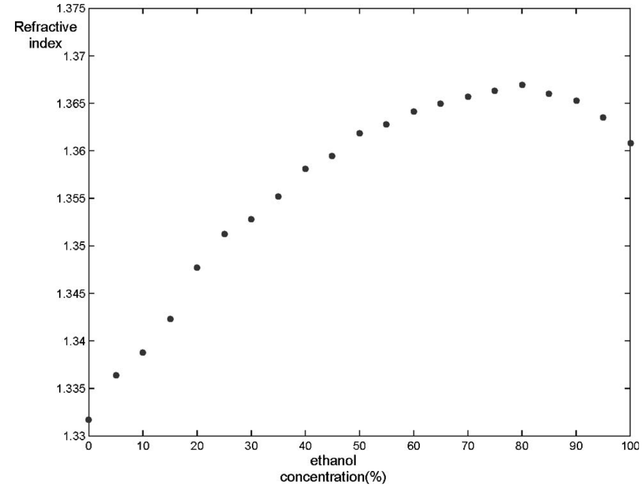

Fig. 5. Refractive index of the ethanol–water solution versus ethanol concentration obtained experimentally.

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